Friedrich-Alexander-Universität
Naturwissenschaftliche Fakultät
Department of DATA SCIENCE

Fraunhofer IIS

Master thesis:

Optimized Unitary Synthesis by Mixed-Integer Non-Linear Programming

Topic description

Quantum computing offers an alternative model of computation, which is more powerful than classical computation in some respects. As a result, the arrival of first prototypical quantum computers has triggered a large research effort in quantum computing during the last decade. *Unitary synthesis* is an essential subroutine in the process of compiling quantum algorithms to specific quantum hardware instructions. The elementary building blocks of quantum algorithms are unitary matrices. While a quantum algorithm may contain arbitrary unitaries, the targeted quantum hardware allows only for a finite set of unitaries. Now, the task is to approximate each unitary in the algorithm as a product of unitaries from this finite set.

More precisely, given an arbitrary unitary $U \in \mathcal{U}(N)$, where $\mathcal{U}(N)$ denotes the group of unitary matrices of dimension N, a finite subset $\mathcal{G} \subset \mathcal{U}(N)$, an error tolerance $\varepsilon > 0$ and an integer $d > 0$, we wish to find a sequence $U_1, U_2, \dots, U_d$, $U_i \in \mathcal{G}$, such that

$$\left\| U - \prod_{i=1}^{d} U_i \right\| \leq \varepsilon\,. \tag{1}$$

Here, $\|\cdot\|$ denotes an arbitrary matrix norm. Moreover, the finite set of unitaries $\mathcal{G}$ is *universal*, which means that such a sequence of unitaries always exists for a sufficiently large d. In practice, some unitaries in $\mathcal{G}$ are more costly to execute on quantum hardware than others. Given a cost associated to each unitary in $\mathcal{G}$, the unitary synthesis problem becomes a combinatorial optimization problem: We aim to find a gate sequence of minimal cost and satisfying (1). For the case $\varepsilon = 0$, the problem is called *exact unitary synthesis* and there exist globally optimal approaches based on search algorithms [1], integer programming [2] and non-linear programming [3] as well as heuristics, for example reinforcement learning [4]. For the case $\epsilon > 0$, much less methods have been developed since the seminal work of Solovay and Kitaev [5].

The goal of this thesis is to develop, analyze, implement and evaluate exact mixed-integer (non-)linear programming methods for unitary synthesis.

Preknowledge in quantum computing is not required, however familiarity with integer programming is necessary.

This thesis will be supervised jointly by the the Department of Data Science (Frauke Liers) and the Fraunhofer IIS.

Contact

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In case of interest please send an email including a Transcript of Records, a short Letter of Motivation as well as a preferred starting date.

References

[1] M. Amy, D. Maslov, M. Mosca, and M. Roetteler. A meet-in-the-middle algorithm for fast synthesis of depth-optimal quantum circuits. *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, 32(6):818–830, June 2013. doi:10.1109/tcad.2013.2244643.

[2] Harsha Nagarajan, Owen Lockwood, and Carleton Coffrin. Quantumcircuitopt: An open-source framework for provably optimal quantum circuit design. In *2021 IEEE/ACM Second International Workshop on Quantum Computing Software (QCS)*, pages 55–63, 2021. doi:10.1109/QCS54837.2021.00010.

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[5] Christopher M. Dawson and Michael A. Nielsen. The solovay-kitaev algorithm, 2005. arXiv:quant-ph/0505030.